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by Thomas M. Grace and Eugene A. Krejsa

Lewis Research Center
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SUMMARY

A study was made of instabilities occurring in a forced-flow, electrically heated, downflow boiler with constant exit pressure. Steady-state conditions and the frequency of oscillation were measured at the onset of oscillations for various feedline characteristics. A model representing the dominant dynamic characteristics of the boiler is presented. This model was verified by the use of impedance concepts, known characteristics of the feed system, and experimental data.

INTRODUCTION

Recent emphasis on more efficient and compact methods of heat transfer has led to the use of forced convective-boiling systems. One of the more serious problems encountered with these systems is the various types of unstable behavior that occur. Instabilities inevitably lead to degraded performance and may become serious enough to cause actual failure of the system. It is, therefore, critically important to understand boiling-loop instability.

There are a great number of references that discuss experimentally observed boiler instabilities. Jeglic and Grace (ref. 1) observed flow oscillations in the subcooled region. They found that the onset of these oscillations was strongly dependent on the thermodynamic conditions existing in the boiler. In reference 2, Lowdermilk reports that oscillations could greatly affect critical heat flux. He found that the instability could be prevented by restricting the flow upstream of the test section and that the presence of a compressible volume in the feedline had a pronounced destabilizing effect. Aladyev and his associates report similar results in reference 3. It has also been reported that restricting the boiler exit has a destabilizing effect. Thus, it appears that boiler-loop instabilities are caused by an interaction between the feed-system, boiler, and exit hardware.

The dynamic characteristics of the feed-system, boiler, and exit hardware must be known in order to describe this interaction. Since little is known about the boiler dynamic characteristics, a study was initiated at the NASA Lewis Research Center on a boiler facility that has a well-defined feed-system and boiler-exit condition. The facility was fabricated so that various dynamic characteristics of the feed system could be obtained and the pressure at the boiler exit would tend to be constant. Thus, for this facility, the instabilities were caused only by feed-system - boiler interaction.

It is necessary to determine how the feed system and the boiler interact in order to make a quantitative analysis of this type of instability. The feed system supplies flow to the boiler, and this flow is controlled in part by the pressure at the boiler inlet. Thus, the interaction that is a characteristic of this instability is a pressure-flow interaction between the feed system and the boiler. Therefore, the characteristics needed to analyze the stability of this system are the boiler and feedline impedances. The boiler impedance gives the dynamic relation between pressure and flow at the boiler inlet as a function of frequency and boiler parameters. The feedline impedance gives the dynamic relation between pressure and flow at the boiler inlet as a function of frequency and feedline parameters. The feedline impedance can be calculated by using line-dynamics theory, but the boiler impedance is unknown. This report presents a model for the boiler impedance and verifies the fact that this model represents the dominant characteristics of the boiler over the range of investigation.

ANALYSIS OF BOILER DYNAMICS

If it is assumed that the boiler pressure drop is a function of total flow at the boiler inlet and vapor flow at the boiler outlet, then

$$(P_b)_{in} - (P_b)_{out} = \delta P_b\left[W_t, (W_v)_{out}\right]$$

(All symbols are defined in appendix A.) Since $(P_b)_{out}$ is constant, the impedance of the boiler can be written as

$$Z_b = \frac{(\Delta P_b)_{in}}{\Delta W_t} = \left.\frac{\partial \delta P_b}{\partial W_t}\right|_{(W_v)_{out}} + \left.\frac{\partial \delta P_b}{\partial (W_v)_{out}}\right|_{W_t} \frac{(\Delta W_v)_{out}}{\Delta W_t}$$

For a constant-diameter boiler, the vapor generation rate per unit length is independent of total flow and depends only on the heat flux and fluid properties. Thus,

if fluid-property changes are neglected, the vapor flow at the boiler outlet depends only on the heat flux and the length of the two-phase region. For electrical heating and thin-wall tubing, the heat flux is constant. Thus, changes in the vapor flow at the boiler outlet are produced only by changes in the length of the two-phase region:

$$(\Delta W_v)_{out} = \frac{\pi D q}{\lambda} \Delta L_{tp}$$

Since the total heated length is constant, changes in the length of the two-phase region are produced by changes in the length of the subcooled region:

$$\Delta L_{tp} = -\Delta L_{sc}$$

Thus, the boiler impedance is given by

$$Z_b = \frac{(\Delta P_b)_{in}}{\Delta W_t} = \left.\frac{\partial \delta P_b}{\partial W_t}\right|_{(W_v)_{out}} - \frac{\pi D q}{\lambda} \left.\frac{\partial \delta P_b}{\partial (W_v)_{out}}\right|_{W_t} \frac{\Delta L_{sc}}{\Delta W_t}$$

Solving the subcooled-region energy equation with constant heat flux assumed gives

$$\frac{\Delta L_{sc}}{\Delta W_t} = \frac{C_p(T_{sat} - T_{in})}{\pi D q}\left(\frac{1 - e^{-\tau s}}{\tau s}\right)$$

where

$$\tau = \frac{\rho_l C_p A_t (T_{sat} - T_{in})}{\pi D q}$$

is the time required for a particle to travel from the boiler inlet to the end of the subcooled region, and s is the Laplace variable. (The details of this solution are given in appendix B.) Thus,

$$Z_b = \frac{(\Delta P_b)_{in}}{\Delta W_t} = \left.\frac{\partial \delta P_b}{\partial W_t}\right|_{(W_v)_{out}} - N_{sc} \left.\frac{\partial \delta P_b}{\partial (W_v)_{out}}\right|_{W_t} \left(\frac{1 - e^{-\tau s}}{\tau s}\right)$$

where

$$N_{sc} = \frac{C_p(T_{sat} - T_{in})}{\lambda}$$

At the neutral stability point $s = j\omega$, therefore, the boiler impedance at neutral stability is given by

$$Z_b = \frac{(\Delta P_b)_{in}}{\Delta W_t} = \left.\frac{\partial \delta P_b}{\partial W_t}\right|_{(W_v)_{out}} - N_{sc} \left.\frac{\partial \delta P_b}{\partial (W_v)_{out}}\right|_{W_t} \left(\frac{1 - e^{-j\omega\tau}}{j\omega\tau}\right)$$

This expression describes the change in boiler inlet pressure, caused by a change in total flow, as a function of boiler steady-state characteristics and the frequency of oscillation. The frequency dependency is contained in the term $(1 - e^{-j\omega\tau})/j\omega\tau$. The amplitude and phase of this term are plotted as a function of $\omega\tau$ in figure 1. For the data obtained in this study, the term $N_{sc}\left[\partial \delta P_b/\partial (W_v)_{out}\right]_{W_t}$ $(1 - e^{-j\omega\tau})/j\omega\tau$ tended to be larger than $(\partial \delta P_b/\partial W_t)_{(W_v)_{out}}$. Under these circumstances, the model indicates that the

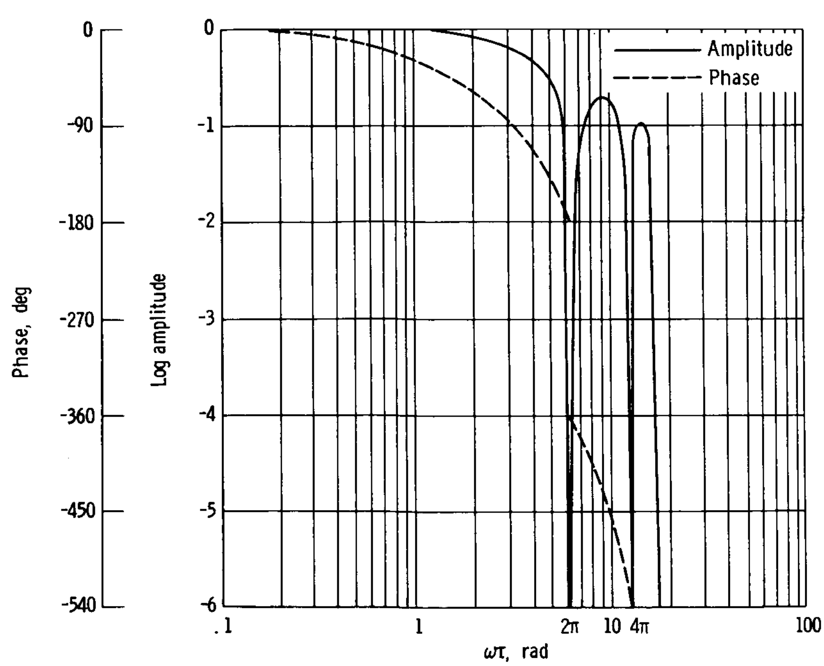


Figure 1. - Amplitude and phase of frequency dependent term in model for boiler inlet impedance.

motion of the interface between the subcooled and two-phase regions tends to play a dominant role.

APPARATUS AND PROCEDURE

Description of Apparatus

The facility used in this study is a closed-loop, low-pressure, forced-flow, convective-boiling system. Water flowed vertically down through the boiler. A schematic diagram of the facility is given in figure 2. The test section (boiler) was constructed of 0.00167-foot-thick-wall tubing with a 0.0208-foot outside diameter. The heated length of the test section was 1.75 feet (100 diam) and was preceded by a 15-diameter unheated length to reduce the hydrodynamic entrance effects. Heat was supplied to the fluid by electrical resistance heating of the tube. Current (ac) was supplied to the test section from a saturable reactor controlled by a Variac. A special thermocouple was fitted 0.0208 foot from the exit of the tube as a burnout detector. The power to the test section would be shut off automatically if this thermocouple indicated

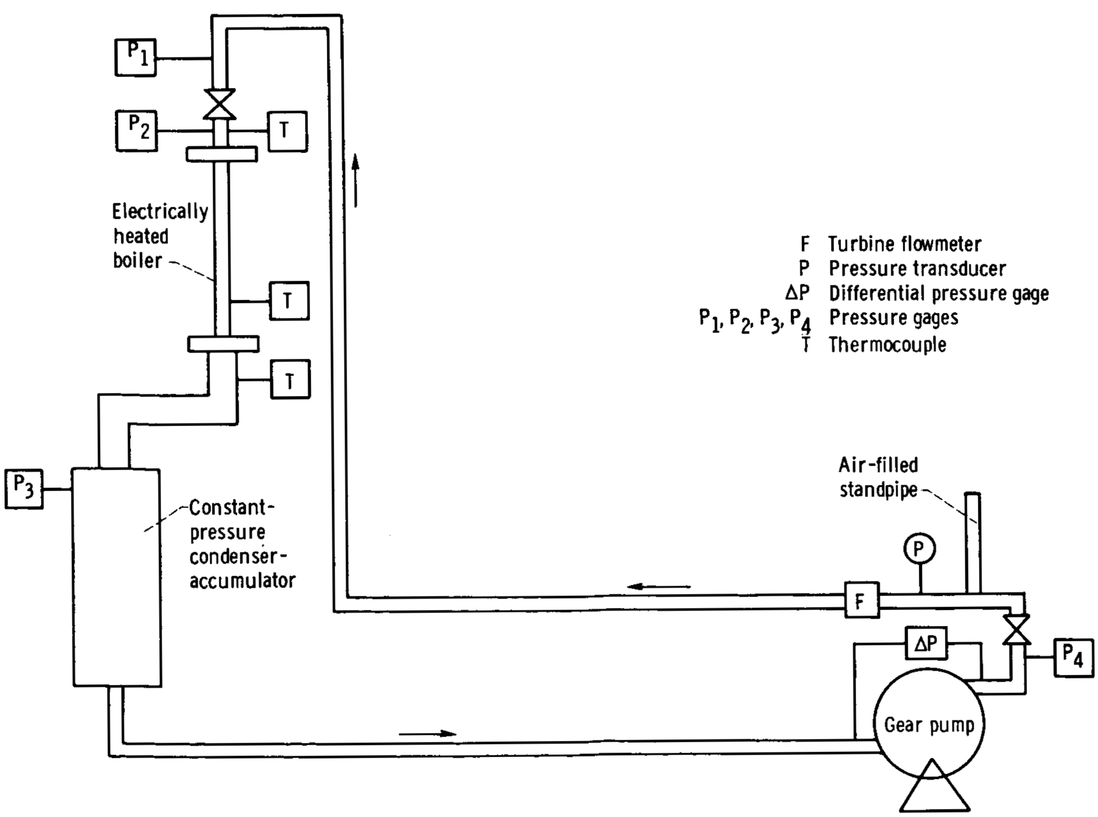


Figure 2. - Schematic diagram of test facility.

that the wall temperature exceeded a preset value.

Flow is provided by a gear pump powered by a variable-speed drive. In addition, a needle valve was located just downstream of the pump to increase the slope of the effective head-flow curve of the pump. This valve was followed by any one of 10 air-filled standpipes that acted as an accumulator or hydraulic capacitance. The standpipe was teed into the line a short distance downstream of the pump. Following this is a 0.0417-foot-diameter line leading to a remote controlled globe valve. This valve was used to provide variable flow resistance at the inlet to the test section. Provisions were made to bleed off the air that would tend to become trapped at the high point in the system.

A condenser-accumulator, a shell and tube heat exchanger, was located immediately downstream of the boiler. It served a dual purpose of condensing any vapor produced by the boiler and holding the boiler exit pressure constant. The loop was filled in such a manner that, when flow was established, a fairly large volume of air was trapped on the shell side of the heat exchanger, into which the steam and hot water from the boiler were ejected. The water to cool this condenser-accumulator passed through the tubes. The air in the condenser held the boiler exit pressure fairly constant. While the boiler exit pressure tended to rise as the temperature of the boiler effluent was raised, it rose quite slowly and did not change significantly as saturation conditions were approached. From all indications, the boiler exit pressure appeared to be insensitive to perturbations. An additional heat exchanger, with the test fluid circulating through the tubes, was used to cool the fluid to a temperature between 60° and 80° F. The temperature was determined by the prevailing cooling-tower water temperature.

Instrumentation

The instrumentation that was included in this study is shown in the schematic diagram of the apparatus (fig. 2). Pressure was measured with 0- to 14 400-pound-per-square-foot range Bourdon gages. There were four basic steady-state pressure measurements: P_1, just upstream of the globe valve; P_2, at the boiler inlet; P_3, at the condenser-accumulator; and P_4, at the pump outlet. The pressure drop across components was determined by taking the differences, both in the lines and the system, and also accounting for fluid friction in the lines. All the gages had small valves in the lines to damp out oscillations and fluctuations, but the valve on the condenser-accumulator was always left open. In addition, great care was taken to ensure that the lines for P_1, P_2, and P_4 were always completely filled with liquid by filling the system under vacuum. These lines were then always valved off when flow was not established. They would remain shut until flow was circulating in the loop and be

closed again before shutting off the pump. This procedure was most critical for gages P_1 and P_2, which were in the portion of the facility that would fill with air when the rig was shut down. A differential gage was used to measure the pressure drop across the pump, but only for ease of operation and as a cross-check on the other measurements. A strain-gage-type pressure transducer was mounted in the line about 3 feet downstream from the standpipe. It was used to measure pressure oscillations.

Flow was measured with a turbine flowmeter located just downstream of the pressure transducer. The pulsed output of this meter was read directly on a counter to provide the basic steady-state flow measurement. In addition, the pulsed output was converted to direct current. The converted output was read on a dial meter and recorded on a pen recorder.

Thermocouples were used to measure the temperature of the fluid entering and leaving the test section. These temperatures were read on a multipoint recorder. In addition, the burnout-detector-thermocouple reading was continuously recorded on a stripchart recorder.

The electrical power supplied to the boiler was measured with a wattmeter. The speed at which the pump was rotating was indicated on a dial meter.

The oscillatory or dynamic data consisted of the flow and pressure measurements located slightly downstream of the standpipe. The pressure measurement was taken from the strain-gage transducer, and the flow measurement was taken from the output of the frequency converter. Both these signals were recorded on a two-channel pen recorder. It was possible to amplify both these signals and to eliminate partially the direct-current level on the signals. Thus, the oscillatory portion of each signal could be played over a wider scale range and, thus, be better examined. A timer fed pulses at 1-second intervals to the recorder to provide a time scale for the dynamic data. From these measurements, it was possible to determine the onset and the extent of oscillatory behavior and the frequency of oscillations.

Experimental Procedure

In the course of this program, instability was best approached by increasing the power supplied to the boiler. This method was used to obtain the data and gave a fairly sharp onset of instability. It was also the method that gave the best control for making the small changes needed in approaching the unstable condition. Thus, at any particular feedline and mean flow condition, instability was approached by increasing the power to the test section.

Ten different capped-off stainless-steel-tubing standpipes were used as accumulators. The standpipes spanned a volume range of 8.82×10^{-5} to 1.62×10^{-2} cubic feet.

For each accumulator, a series of globe-valve resistance settings were run at three different mean flow rates: 1.08×10^{-3}, 2.07×10^{-3}, and 3.06×10^{-3} slug per second.

For a given standpipe, the following procedure was used to obtain a data point: The pump was turned on to establish flow in the loop, and the valves on gages P_1 and P_2 were opened. Air pockets were removed from the system. The desired mean flow rate was then established, and the pump valve was set to take approximately a 5000-pounds-per-square-foot drop across it. Then, the globe valve was set to take the desired pressure drop across it. At this point, a mean flow rate and a feedline condition were established, and power was supplied to the boiler. The power was increased in increments and the steady-state data (pressures, power, flow, temperatures, etc.) were taken at appropriate intervals. The power was incrementally increased until either an oscillation set in or the current limitation on the wattmeter was reached. The onset of the oscillations could be observed by examining the pressure and flow signals on the pen recorder. The oscillatory signals were recorded for a sufficient length of time to establish the frequency of oscillation. Often, however, the oscillation would trigger a burnout and automatically shut off the power, sometimes after only a few periods. After obtaining a data point in this manner, a new condition would be established and the procedure repeated.

DATA REDUCTION

For a system at neutral stability, the total impedance around the loop must be zero at the frequency of oscillation. For this system, the total impedance around the loop is the sum of the feedline impedance and the boiler impedance. Thus, at neutral stability, the negative of the boiler impedance must equal the feedline impedance. This fact can be used to check the validity of the model used to obtain the boiler impedance.

An expression for the feedline impedance can be obtained by substituting the appropriate electrical analogies for the feed-system components. Resistance is analogous to pressure-drop devices, such as pumps and valves. Capacitance is analogous to compressible volume, and inductance is analogous to fluid inertia. The electrical analog of the feed system used in this study is shown in figure 3. The inertia of the fluid in the lines from the condenser to the pump L_1 is defined as the sum of the

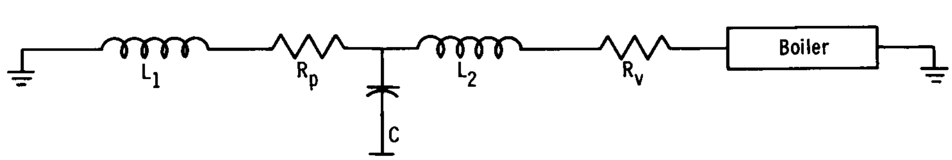


Figure 3. - Electrical analog of feed system.

ratios of length to area for these lines. The combined resistance of the pump and the needle valve just downstream of the pump R_p is the sum of the slopes of pressure drop against flow curves for these elements. With air assumed to act as an ideal, isothermal gas, the capacitance of the air in the standpipe C is defined as

$$C = \frac{\rho_l V_o}{P_o}\left(\frac{P_o}{P_g}\right)^2$$

where V_o is the initial volume of air at atmospheric pressure P_o, and P_g is the mean absolute gas pressure at operating conditions. The inertia of the fluid in the lines from the standpipe to the valve at the boiler inlet L_2 is defined as the sum of the ratios of length to area for these lines. The combined resistance of the valve at the boiler inlet and the lines from the standpipe to the boiler inlet R_v is obtained from slopes of pressure drop against flow curves. The feedline impedance is

$$Z_f = R_v + \frac{R_p}{\left(1 - \omega^2 L_1 C\right)^2 + \omega^2 R_p^2 C^2} + j\omega\left[L_2 + \frac{L_1 - C\left(R_p^2 + \omega^2 L_1^2\right)}{\left(1 - \omega^2 L_1 C\right)^2 + \omega^2 R_p^2 C^2}\right]$$

The amplitude and phase of the feedline impedance can be calculated from the following relations:

$$\text{Amplitude} = \sqrt{(\text{Real})^2 + (\text{Imaginary})^2}$$

$$\text{Phase} = \tan^{-1}\left(\frac{\text{Imaginary}}{\text{Real}}\right)$$

The boiler impedance is

$$Z_b = \left.\frac{\partial \delta P_b}{\partial W_t}\right|_{(W_v)_{out}} - N_{sc}\left.\frac{\partial \delta P_b}{\partial (W_v)_{out}}\right|_{W_t}\left(\frac{1 - e^{-j\omega\tau}}{j\omega\tau}\right)$$

where

$$N_{sc} = \frac{C_p(T_{sat} - T_{in})}{\lambda}$$

$$\tau = \frac{\rho_l C_p A_t (T_{sat} - T_{in})}{\pi D q}$$

and where $(\partial \delta P_b / \partial W_t)_{(W_v)_{out}}$ is the slope of boiler pressure drop against total flow at constant exit vapor flow, $\left[\partial \delta P_b / \partial (W_v)_{out}\right]_{W_t}$ is the slope of boiler pressure drop against exit vapor flow at constant total flow, and ω is the observed frequency of oscillation. The amplitude and phase of the negative of the boiler impedance are obtained from the following relations:

$$\text{Amplitude} = \sqrt{(\text{Real})^2 + (\text{Imaginary})^2}$$

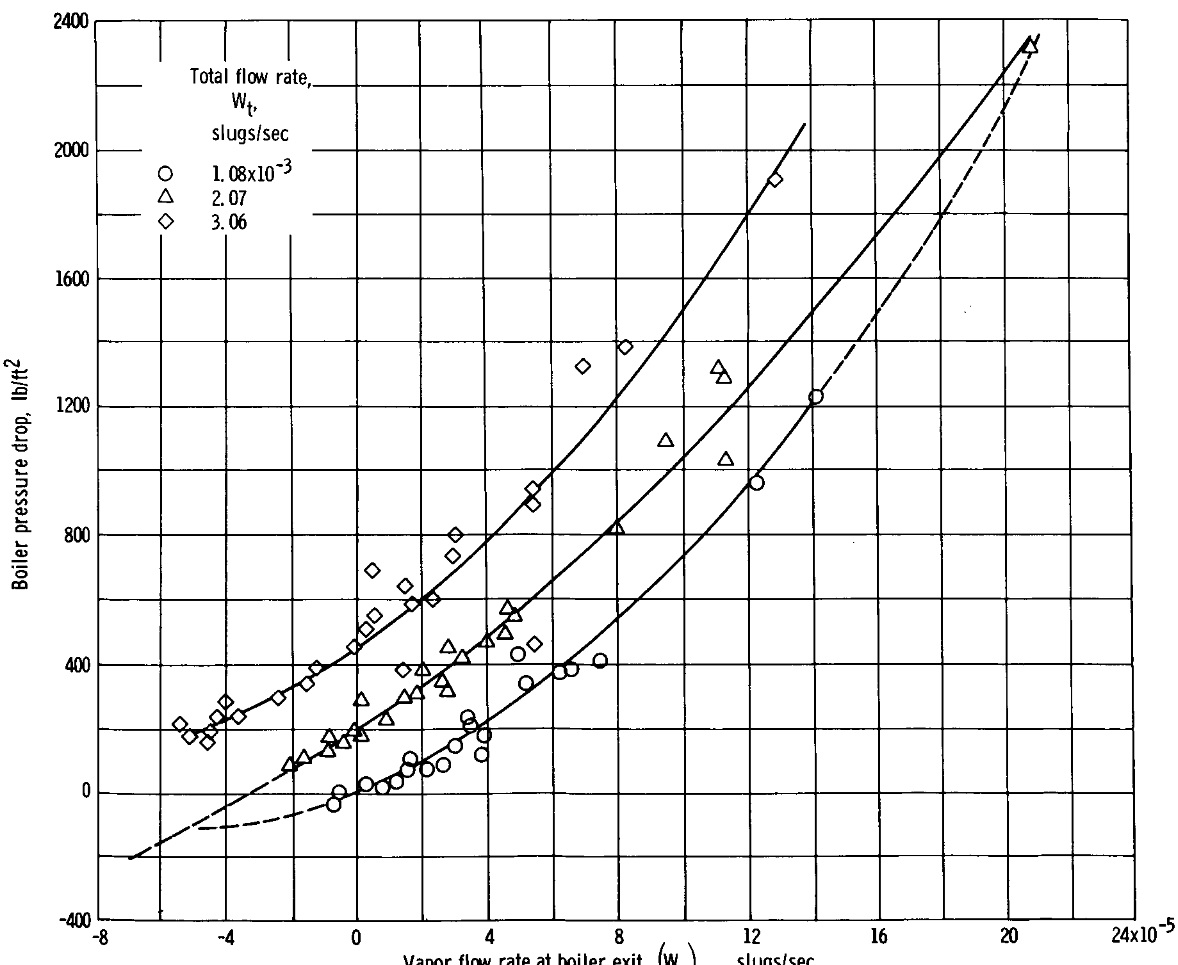
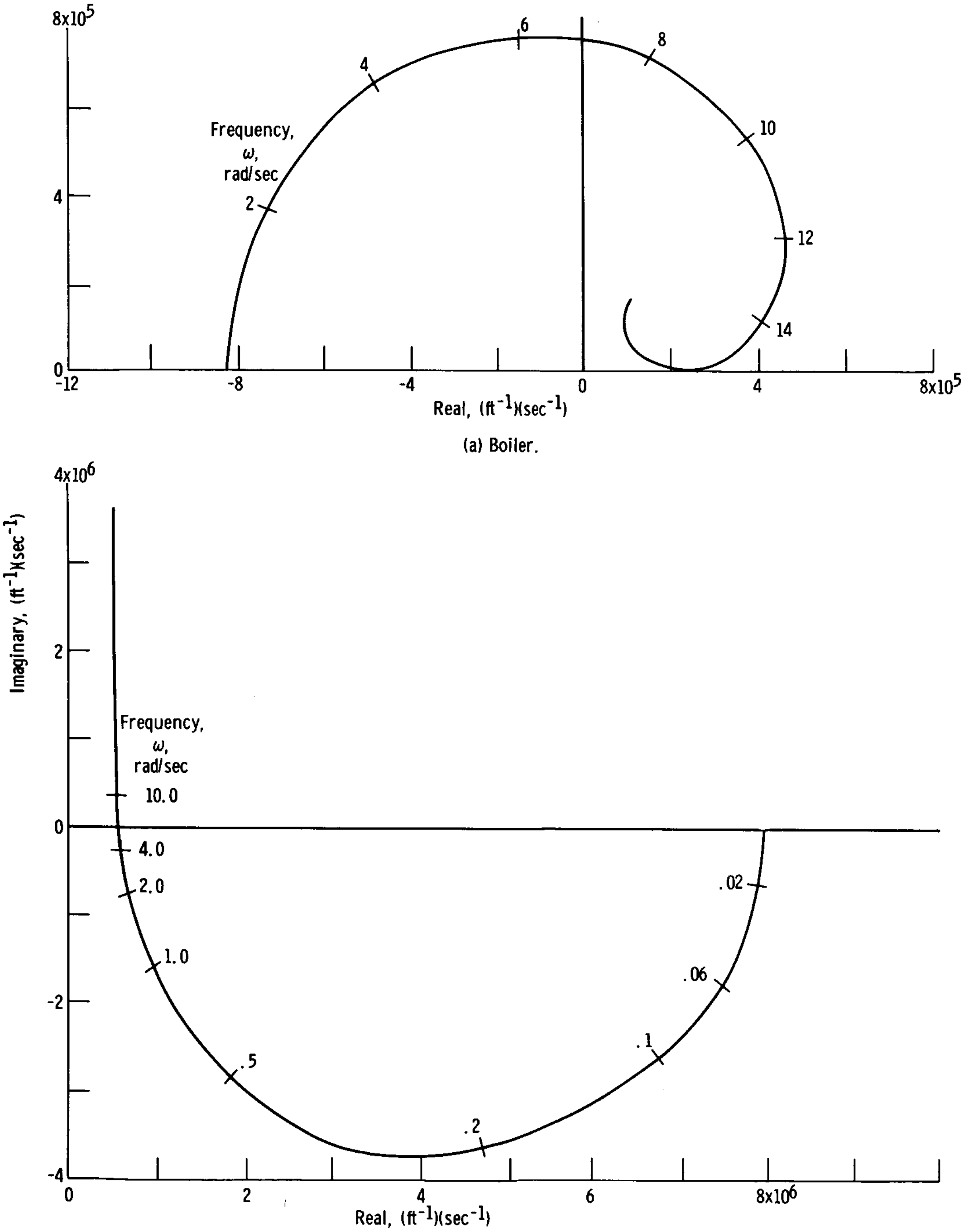


Figure 4. - Steady-state boiler pressure drop against vapor flow at boiler exit with constant total flow.



(a) Boiler.

(b) Feedline.

Figure 5. - Typical plot of impedance in complex plane. Run 794.

$$\text{Phase} = \tan^{-1}\left(\frac{\text{Imaginary}}{\text{Real}}\right) - 180^{\circ}$$

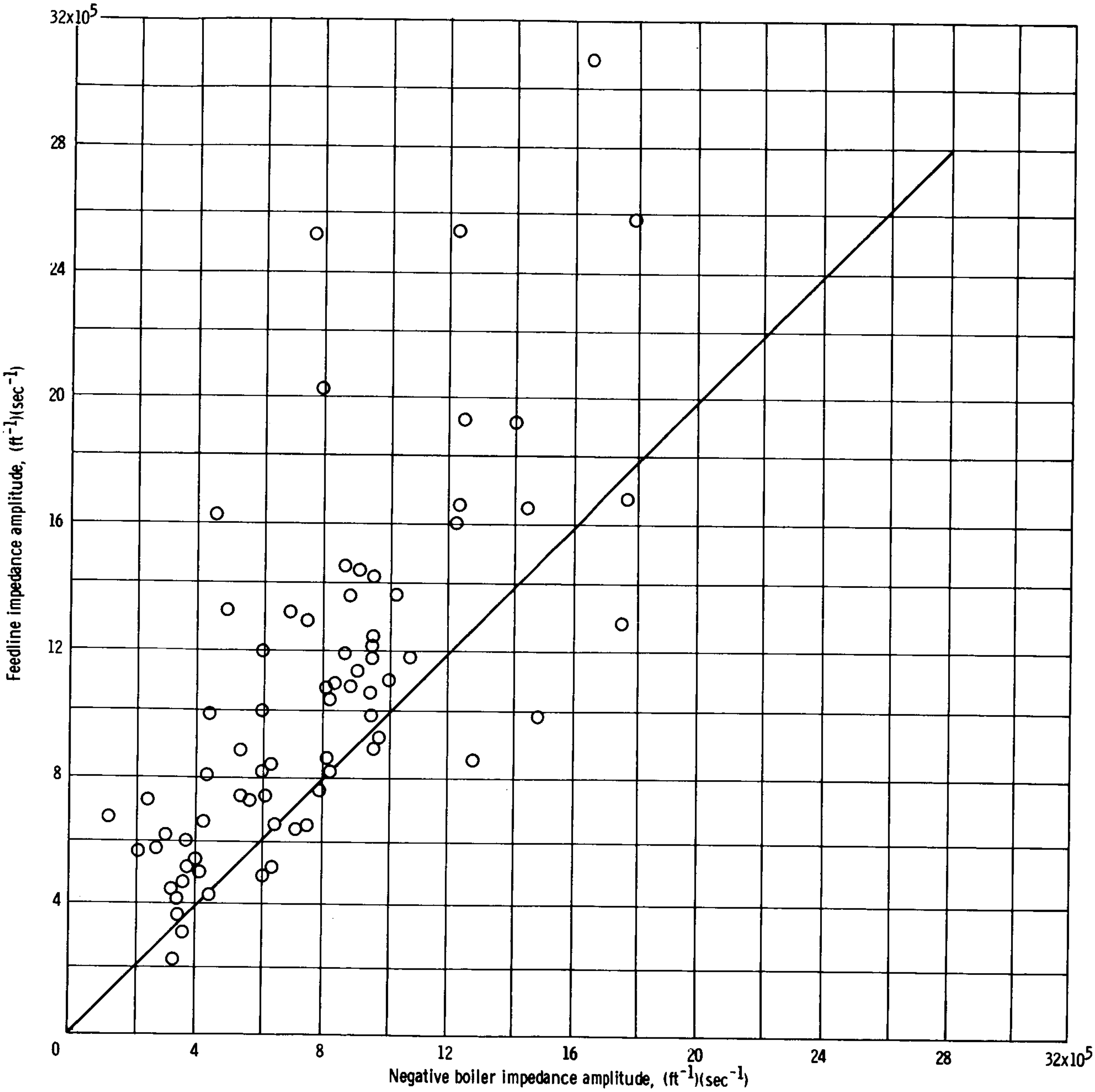
Plots of boiler pressure drop against exit vapor flow at constant total flow are presented in figure 4. The vapor flow used in these plots was calculated assuming thermal equilibrium. These plots were used to evaluate the two partial derivatives needed to calculate the boiler impedance. All quantities in both the boiler and the feedline impedances are evaluated at the observed inceptions of instability. A tabulation of the reduced data for each observed inception of instability is given in tables I and II. Figure 5 shows typical plots of the boiler and feedline impedances in the complex plane.

RESULTS AND DISCUSSION

The feedline impedance, which was determined from known characteristics and steady-state data at the inception of instability, is presented in table I for each observed point of instability. The negative of the boiler impedance, which was determined from the model of boiler dynamic characteristics and steady-state data, is presented in table II for each point of instability. Since the sum of the boiler and feedline impedances must be zero at neutral stability, the amplitude and phase of the feedline impedance should agree with the amplitude and phase of the negative of the boiler impedance if the boiler model is valid.

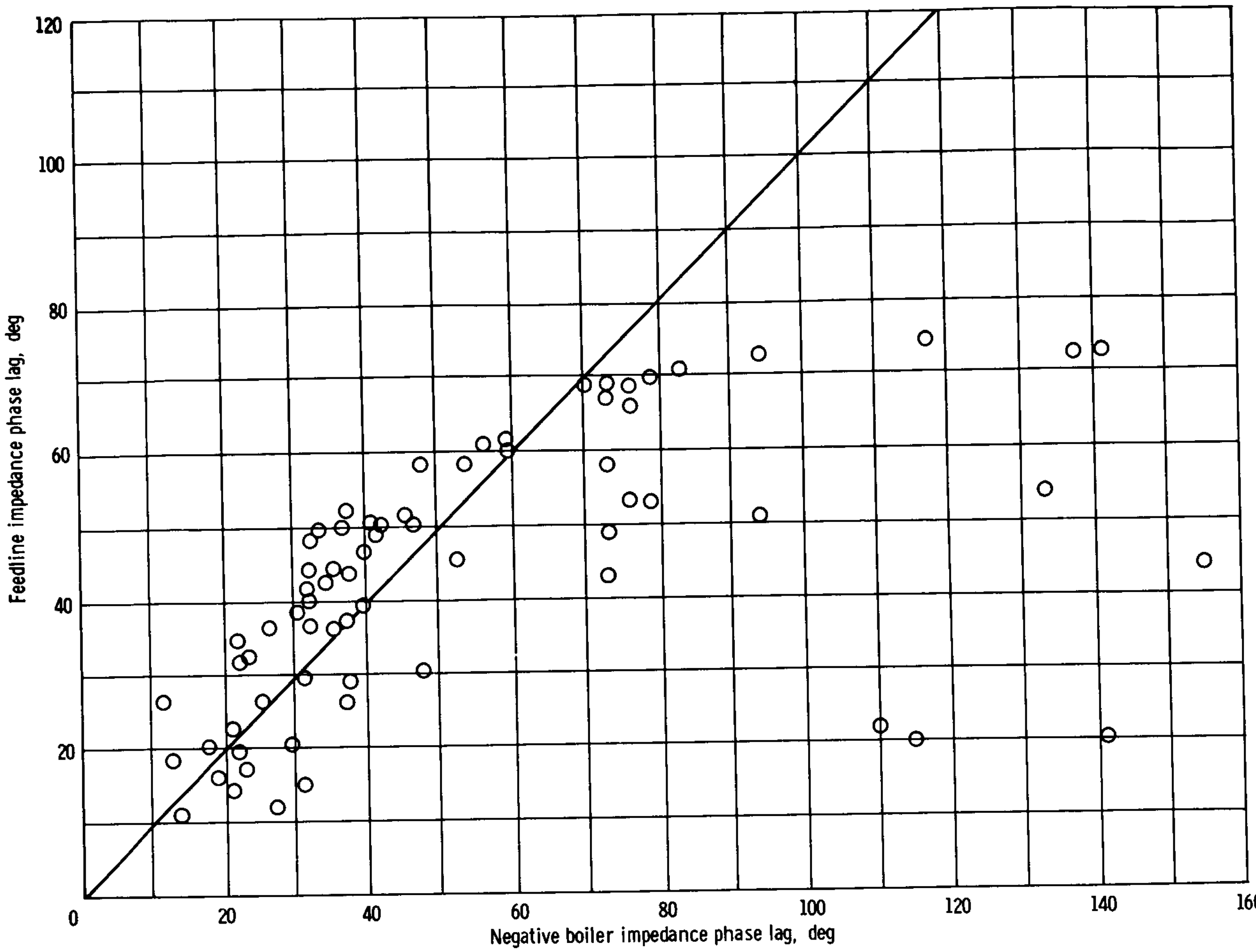
The amplitude of the boiler impedance is plotted against the amplitude of the feedline impedance in figure 6(a). The phase of the negative of the boiler impedance is plotted against the phase of the feedline impedance in figure 6(b). The solid lines were drawn at 45° to facilitate comparison of the two quantities. Most of the amplitudes and phases of the boiler impedance agree within ±50 percent of the amplitudes and phases of the feedline impedance. Thus, the data show that the model, while not completely describing the boiler, does describe its dominant characteristics.

While some of the scatter in figure 6 is caused by the evaluation of both the feedline and boiler impedances from experimental data, the majority of the scatter is probably caused by the assumptions made when the boiler model was derived. Energy storage in the tube wall and mass and energy storage in the two-phase region were neglected. Although the effect of these assumptions is negligible at low frequencies, it can become significant at higher frequencies. In addition to storage, the effects of subcooled boiling were neglected. The existence of subcooled boiling should be taken into account when the partial derivatives of boiler pressure drop are evaluated. That is, the actual vapor flow, rather than the vapor flow calculated assuming thermal equilibrium, should be used when evaluating these partials. Subcooled boiling will also affect the change in



(a) Amplitude.

Figure 6. - Comparison of feedline impedance and negative boiler impedance.



(b) Phase lag.

Figure 6. - Concluded.

vapor flow at the boiler exit due to a change in the two-phase length and will affect the value of τ. The model used in this report defines τ as the time required for a particle to travel from the boiler inlet to the point where the bulk fluid temperature is equal to saturation temperature. In the presence of subcooled boiling, τ should be defined as the time required for a particle to travel from the boiler inlet to the point where boiling begins.

CONCLUDING REMARKS

The analysis of boiler dynamic characteristics presented in this report yields a simple model that agrees reasonably well with experimental data. This model shows

that for the conditions of this study, the motion of the interface between the subcooled and two-phase regions played a dominant role.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 16, 1966,
120-27-04-27-22.

APPENDIX A

SYMBOLS

A_t	cross-sectional area of boiler tube, ft^2
C	capacitance of air in standpipe, (ft)(sec^2)
C_p	heat capacity of liquid, Btu/(slug)(^oF)
D	diameter of boiler tube, ft
L_{sc}	length of subcooled region in boiler, ft
L_{tp}	length of two-phase region in boiler, ft
L_1	inertia of fluid in line between condenser and pump, ft^{-1}
L_2	inertia of fluid in line between standpipe and boiler, ft^{-1}
N_{sc}	$C_p(T_{sat} - T_{in})/\lambda$
$(P_b)_{in}$	pressure at boiler inlet, lb/ft^2
$(P_b)_{out}$	pressure at boiler outlet, lb/ft^2
P_g	absolute gas pressure at standpipe, lb/ft^2
P_o	initial pressure of gas in standpipe, lb/ft^2
P_1	pressure just upstream of globe valve, lb/ft^2
P_2	pressure at boiler inlet, lb/ft^2
P_3	pressure at condenser-accumulator, lb/ft^2
P_4	pump outlet pressure, lb/ft^2
q	heat flux, Btu/(ft^2)(sec)
R_p	resistance of pump and needle valve downstream of pump, (ft^{-1})(sec^{-1})
R_v	resistance of globe valve and line from standpipe to boiler inlet, (ft^{-1})(sec^{-1})
s	Laplace variable, sec^{-1}
t	time, sec
T_l	liquid temperature, ^oF
T_{in}	liquid temperature at boiler inlet, ^oF
T_{sat}	saturation temperature, ^oF
V_o	initial volume of gas in standpipe, ft^3
W_t	total flow rate at boiler inlet, slug/sec
$(W_v)_{out}$	vapor flow rate at boiler outlet, slugs/sec
X	position, ft
Z_b	boiler impedance, (ft^{-1})(sec^{-1})
Z_f	feedline impedance, (ft^{-1})(sec^{-1})
δP_b	boiler pressure drop, lb/ft^2
λ	heat of vaporization, Btu/slug
ρ_l	liquid density, $slugs/ft^3$
τ	subcooled dead time, sec
ω	frequency, rad/sec

APPENDIX B

DERIVATION OF DYNAMIC CHARACTERISTICS OF SUBCOOLED REGION

The energy equation for the subcooled region with constant heat flux and constant density is

$$W_t C_p \frac{\partial T_l}{\partial X} + \rho_l A_t C_p \frac{\partial T_l}{\partial t} = \pi D q \tag{B1}$$

or

$$\frac{\partial T_l}{\partial X} + \frac{\rho_l A_t}{W_t} \frac{\partial T_l}{\partial t} = \frac{\pi D q}{W_t C_p} \tag{B2}$$

Putting this equation in perturbation form yields

$$\frac{\partial \Delta T_l}{\partial X} + \frac{\rho_l A_t}{W_t} \frac{\partial \Delta T_l}{\partial t} = \frac{-\pi D q}{C_p W_t^2} \Delta W_t \tag{B3}$$

Taking the Laplace transform gives

$$\frac{d\Delta T_l}{dX} + \frac{\rho_l A_t s}{W_t} \Delta T_l = - \frac{\pi D q}{C_p W_t^2} \Delta W_t \tag{B4}$$

Integrating gives

$$\Delta T_l = C\exp\left(- \frac{\rho_l A_t s}{W_t} X\right) - \frac{\pi D q}{C_p \rho_l A_t s} \frac{\Delta W_t}{W_t} \tag{B5}$$

at $X = 0$; $\Delta T_l = 0$. Therefore,

$$C = \frac{\pi D q}{C_p \rho_l A_t s} \frac{\Delta W_t}{W_t} \tag{B6}$$

and

$$\Delta T_l = \left[\exp\left(-\frac{\rho_l A_t s}{W_t} X\right) - 1\right] \frac{\pi D q}{C_p \rho_l A_t s} \frac{\Delta W_t}{W_t} \tag{B7}$$

Equation (B7) gives the liquid-temperature perturbation at any particular position X as a function of the flow perturbation. What is of interest is the change in the subcooled length. The change in the subcooled length equals the change in the position at which $T_l = T_{sat}$. This change, in turn, can be related to the change in temperature at $X = L_{sc}$ through the steady-state temperature profile if second-order effects are neglected:

$$\Delta L_{sc} = -\frac{\Delta T_l \Big|_{L_{sc}}}{\dfrac{dT_l}{dx}} = -\frac{W_t C_p}{\pi D q} \Delta T_l \Big|_{L_{sc}} \tag{B8}$$

Combining equations (B8) and (B7) gives

$$\Delta L_{sc} = \left[\frac{1 - \exp\left(-\dfrac{\rho_l A_t s}{W_t} L_{sc}\right)}{\dfrac{\rho_l A_t s}{W_t}}\right] \frac{\Delta W_t}{W_t} \tag{B9}$$

or

$$\frac{\Delta L_{sc}}{L_{sc}} = \left(\frac{1 - e^{-\tau s}}{\tau s}\right) \frac{\Delta W_t}{W_t} \tag{B10}$$

where

$$\tau = \frac{\rho_l A_t L_{sc}}{W_t} = \frac{\rho_l C_p A_t (T_{sat} - T_{in})}{\pi D q} \tag{B11}$$

Since

$$\pi D q L_{sc} = W_t C_p (T_{sat} - T_{in})$$

$$\frac{\Delta L_{sc}}{\Delta W_t} = \frac{C_p(T_{sat} - T_{in})}{\pi D q} \left(\frac{1 - e^{-\tau s}}{\tau s}\right)$$

REFERENCES

1. Jeglic, Frank A.; and Grace, Thomas M.: Onset of Flow Oscillations in Forced-Flow Subcooled Boiling. NASA TN D-2821, 1965.
2. Lowdermilk, Warren H.: Lanzo, Chester D.; and Siegel, Byron L.: Investigation of Boiling Burnout and Flow Stability for Water Flowing in Tubes. NACA TN-4382, 1958.
3. Aladyev, I. T.; Miropolsky, Z. L.; Doroshchuk, V. E.; and Styrikovich, M. A.: Boiling Crisis in Tubes. International Developments in Heat Transfer. ASME, 1963, pp. 237-243.

TABLE I. - AMPLITUDE AND PHASE OF FEEDLINE IMPEDANCE AND VALUES NEEDED TO COMPUTE THIS IMPEDANCE

Run	Frequency, ω, rad/sec	Resistance, $(ft^{-1})(sec^{-1})$		Inertia of fluid, ft^{-1}		Capacitance in standpipe, C, $(ft)(sec^2)$	Feedline impedance	
		Pump, R_p	Globe value, R_v	L_1	L_2		Amplitude, $(ft^{-1})(sec^{-1})$	Phase lag, deg
Liquid flow rate at boiler inlet, W_t, 1.08×10^{-3} slug/sec								
470	14.3	11.11×10^6	1.20×10^5	2796	11 320	0.96×10^{-7}	5.76×10^5	73
471	13.2	11.07	1.22			1.08	5.71	73
473	4.65	10.59	1.25			.82	25.2	73
543	16.0	11.33	1.20			1.70	2.28	54
551	16.3	11.70	2.81			2.11	3.07	20
554	3.21	10.68	2.81			1.92	16.3	71
559	20.0	10.23	3.88			1.34	4.26	20
561	26.6	10.49	9.55			.75	10.0	12
584	4.25	10.53	1.25			3.40	6.62	75
598	4.27	10.92	5.72			2.53	10.74	53
690	2.95	10.94	1.26			7.98	4.16	70
695	2.64	11.30	1.75			7.70	5.02	67
700	2.83	10.60	6.52			4.72	10.03	45
762	2.90	11.33	2.08	3120	33 200	4.32	7.45	69
767	2.82	11.47	2.34			4.09	8.25	69
775	2.86	11.18	5.23			3.12	11.94	58
779	3.38	10.52	8.39			2.08	16.6	51
834	2.07	11.07	2.08			.99	4.74	61
837	2.06	11.07	2.32			.96	5.06	60
843	1.90	10.90	4.99			7.43	8.42	50
849	3.21	10.47	9.58			3.46	12.96	37
900	1.40	11.12	2.05			22.04	3.50	52
903	1.44	11.04	3.38			21.16	4.48	39
908	1.51	11.30	8.92			13.58	10.1	26
957	1.12	10.83	9.70			29.5	10.1	15

TABLE I. - Continued. AMPLITUDE AND PHASE OF FEEDLINE IMPEDANCE

AND VALUES NEEDED TO COMPUTE THIS IMPEDANCE

Run	Frequency, ω, rad/sec	Resistance, $(ft^{-1})(sec^{-1})$		Inertia of fluid, ft^{-1}		Capacitance in standpipe, C, $(ft)(sec^2)$	Feedline impedance	
		Pump, R_p	Globe value, R_v	L_1	L_2		Amplitude, $(ft^{-1})(sec^{-1})$	Phase lag, deg
Liquid flow rate at boiler inlet, W_t, 2.07×10^{-3} slug/sec								
510	5.71	6.97×10^6	2.32×10^5	2796	11 320	0.83×10^{-7}	20.3×10^5	66
516	5.56	7.06	2.34			1.33	13.3	69
572	6.03	7.31	2.28			.49	30.8	61
607	5.15	7.38	2.34			4.08	4.92	58
624	4.33	7.47	4.56			2.28	11.1	58
650	2.94	7.27	3.72			6.52	6.35	50
656	2.84	7.47	5.80			5.31	8.95	44
715	2.79	7.23	5.90	3120	33 200	3.38	12.0	52
727	3.16	7.10	7.10			2.68	13.8	50
734	3.05	7.34	11.00			1.52	25.3	48
794	2.62	7.42	5.45			5.93	8.17	43
800	2.64	7.58	7.97			4.33	11.9	41
805	3.00	7.69	10.76			2.93	16.1	39
854	1.65	7.27	3.96			17.14	5.10	36
858	1.60	7.34	5.19			14.99	6.53	34
862	1.65	7.30	6.85			11.77	8.55	32
867	1.91	7.62	9.83			9.30	11.4	26
873	1.92	7.84	17.20			4.44	19.4	22
917	1.26	7.35	6.99			26.1	7.60	20
921	.89	7.42	9.50			21.75	11.0	26
926	1.68	7.57	13.22			15.3	13.8	14
933	1.92	7.72	18.10			8.54	19.3	16
973	.68	7.43	10.18			41.4	10.9	18
977	1.14	7.39	14.00			29.0	14.4	11

TABLE I. - Concluded. AMPLITUDE AND PHASE OF FEEDLINE IMPEDANCE

AND VALUES NEEDED TO COMPUTE THIS IMPEDANCE

Run	Frequency, ω, rad/sec	Resistance, $(ft^{-1})(sec^{-1})$		Inertia of fluid, ft^{-1}		Capacitance in standpipe, C, $(ft)(sec^2)$	Feedline impedance	
		Pump, R_p	Globe value, R_v	L_1	L_2		Amplitude, $(ft^{-1})(sec^{-1})$	Phase lag, deg
Liquid flow rate at boiler inlet, W_t, 3.06×10^{-3} slug/sec								
628	6.16	5.71×10^6	3.54×10^5	2796	11 320	2.75×10^{-7}	6.62×10^5	51
634	6.47	5.87	4.20			2.41	7.45	49
640	5.15	5.98	5.80			1.72	13.32	53
661	3.93	6.06	3.54			6.01	5.38	44
667	3.88	5.99	4.01			5.53	6.05	43
672	3.55	5.99	4.20			4.22	9.15	44
678	3.76	6.28	7.77			3.02	12.24	42
686	4.62	6.50	11.84			2.00	16.91	36
740	4.14	6.16	6.02	3120	33 200	2.32	10.86	46
745	4.14	6.22	6.43			2.09	12.96	49
751	4.33	6.43	7.46			1.54	16.69	50
815	3.22	6.10	5.70			4.55	8.59	41
819	3.21	6.07	7.05			3.91	10.54	40
822	3.31	6.15	8.54			3.48	12.35	38
826	3.98	6.78	16.35			1.42	25.7	36
829	3.14	3.85	5.85			6.83	7.28	29
878	2.17	5.94	5.92			10.41	7.26	30
883	2.17	6.07	6.67			9.80	8.06	29
888	1.89	6.01	9.50			7.59	12.00	31
892	3.27	6.35	13.33			4.88	14.64	20
940	1.50	5.93	6.03			22.2	6.67	22
944	1.65	6.05	8.28			17.7	8.94	19
948	2.28	6.32	13.89			10.03	14.67	14
980	1.01	5.86	5.80			46.2	6.15	17

TABLE II: - AMPLITUDE AND PHASE OF NEGATIVE BOILER IMPEDANCE AND QUANTITIES NEEDED TO CALCULATE THIS IMPEDANCE

Run	Frequency, ω, rad/sec	Total heat input, Btu/sec	Vapor quality at boiler exit	N_{sc}	$\left.\frac{\partial \delta P_b}{\partial W_t}\right\|_{(W_v)_{out}}$, $(ft^{-1})(sec^{-1})$	$\left.\frac{\partial \delta P_b}{\partial (W_v)_{out}}\right\|_{W_t}$, $(ft^{-1})(sec^{-1})$	Subcooled dead time, τ, sec	Negative boiler impedance	
								Amplitude, $(ft^{-1})(sec^{-1})$	Phase lag, deg
Liquid flow rate at boiler inlet, W_t, 1.08×10^{-3} slug/sec									
470	14.3	6.46	0.034	0.158	2.76×10^5	5.88×10^6	0.604	2.71×10^5	137
471	13.2	5.67	.012	.156	2.04	4.32	.669	2.22	141
473	4.65	7.11	.047	.159	3.00	6.72	.543	7.68	94
543	16.0	7.20	.054	.163	3.06	7.20	.550	3.33	133
551	16.3	6.95	.044	.162	3.00	6.48	.567	3.57	141
554	3.21	5.65	.004	.164	1.68	3.84	.700	4.59	83
559	20.0	9.39	.104	.175	2.58	10.56	.453	4.38	115
561	26.6	13.54	.237	.172	-8.51	24.1	.309	14.90	27
584	4.25	6.58	.031	.160	2.70	5.64	.591	6.52	117
598	4.27	7.95	.062	.165	3.12	7.80	.505	9.46	79
690	2.95	4.92	-.006	.147	1.14	3.12	.727	3.38	79
695	2.64	5.03	.004	.147	1.68	3.84	.710	4.10	73
700	2.83	7.47	.070	.152	3.12	8.28	.495	9.46	52
762	2.90	5.81	.025	.148	2.58	5.16	.619	5.41	73
767	2.82	5.97	.029	.148	2.64	5.46	.602	6.06	70
775	2.86	8.03	.084	.151	3.00	9.24	.457	10.80	47
779	3.38	9.91	.116	.153	2.16	11.40	.408	14.48	45
834	2.07	4.98	-.003	.151	1.32	3.30	.738	3.64	59
837	2.06	4.98	-.003	.152	1.32	3.30	.743	3.67	59
843	1.90	6.25	.037	.152	2.82	6.00	.591	6.42	46
849	3.21	10.17	.134	.157	1.26	12.6	.379	17.59	37
900	1.40	4.70	-.014	.154	.60	2.64	.798	3.36	37
903	1.44	4.72	-.013	.154	.72	2.70	.792	3.34	39
908	1.51	6.34	.028	.159	2.64	5.46	.610	6.12	37
957	1.12	5.47	.004	.158	1.68	3.84	.702	4.44	31

TABLE II. - Continued. AMPLITUDE AND PHASE OF NEGATIVE BOILER IMPEDANCE AND QUANTITIES NEEDED TO CALCULATE THIS IMPEDANCE

Run	Frequency, ω, rad/sec	Total heat input, Btu/sec	Vapor quality at boiler exit	N_{sc}	$\left.\frac{\partial \delta P_b}{\partial W_t}\right\|_{(W_v)_{out}}$, $(ft^{-1})(sec^{-1})$	$\left.\frac{\partial \delta P_b}{\partial (W_v)_{out}}\right\|_{W_t}$, $(ft^{-1})(sec^{-1})$	Subcooled dead time, τ, sec	Negative boiler impedance	
								Amplitude, $(ft^{-1})(sec^{-1})$	Phase lag, deg
Liquid flow rate at boiler inlet, W_t, 2.07×10^{-3} slug/sec									
510	5.71	10.22	0.001	0.159	2.06×10^{5}	6.66×10^{6}	0.378	7.87×10^{5}	76
516	5.56	9.54	-.007	.155	1.86	6.12	.395	7.00	76
572	6.03	16.87	.102	.167	6.05	13.56	.241	16.62	56
607	5.15	9.95	-.003	.158	2.04	5.34	.386	6.08	73
624	4.33	11.76	.020	.165	2.88	8.04	.341	10.14	53
650	2.94	9.30	-.010	.156	1.80	5.87	.407	7.24	42
656	2.84	11.10	.016	.158	2.52	7.74	.346	9.60	35
715	2.79	10.72	.004	.162	2.28	6.84	.367	8.68	37
727	3.16	10.59	.013	.155	2.58	7.50	.356	8.94	41
734	3.05	13.70	.057	.155	4.20	10.56	.275	12.20	32
794	2.62	9.97	.005	.152	2.28	6.95	.371	8.25	35
800	2.64	11.26	.024	.152	3.00	8.28	.328	9.56	32
805	3.00	12.96	.047	.155	4.20	10.56	.291	12.17	33
854	1.65	8.34	-.024	.154	1.32	4.98	.449	6.29	26
858	1.60	9.61	.007	.153	1.86	6.12	.387	7.49	22
862	1.65	10.03	.001	.156	2.06	6.66	.387	8.12	23
867	1.91	10.68	.014	.157	2.64	7.56	.358	9.08	25
873	1.92	13.77	.056	.160	4.20	10.43	.283	12.51	21
917	1.26	9.97	-.008	.164	1.86	6.05	.400	7.94	18
921	.89	10.33	-.004	.165	2.04	6.36	.388	8.46	12
926	1.68	11.71	.016	.166	2.52	7.74	.344	10.27	21
933	1.92	14.03	.055	.165	2.94	10.38	.286	14.11	19
973	.68	9.97	-.002	.156	2.04	6.47	.380	8.05	13
977	1.14	11.21	.023	.154	3.00	8.21	.334	9.65	14

TABLE II. - Concluded. AMPLITUDE AND PHASE OF NEGATIVE BOILER IMPEDANCE AND QUANTITIES NEEDED TO CALCULATE THIS IMPEDANCE

Run	Frequency, ω, rad/sec	Total heat input, Btu/sec	Vapor quality at boiler exit	N_{sc}	$\left.\frac{\partial \delta P_b}{\partial W_t}\right\|_{(W_v)_{out}}$, $(ft^{-1})(sec^{-1})$	$\left.\frac{\partial \delta P_b}{\partial (W_v)_{out}}\right\|_{W_t}$, $(ft^{-1})(sec^{-1})$	Subcooled dead time, τ, sec	Negative boiler impedance	
								Amplitude, $(ft^{-1})(sec^{-1})$	Phase lag, deg
Liquid flow rate at boiler inlet, W_t, 3.06×10^{-3} slug/sec									
628	6.16	13.55	-0.012	0.155	4.20×10^{5}	4.32×10^{6}	0.278	4.20×10^{5}	94
634	6.47	14.20	-.005	.156	3.24	5.63	.268	6.16	73
640	5.15	16.11	.008	.163	2.58	5.10	.246	4.72	76
661	3.93	12.20	-.023	.152	6.00	2.04	.303	3.96	155
667	3.88	13.20	-.012	.152	4.20	4.32	.280	3.70	73
672	3.55	15.36	.006	.157	2.70	7.98	.249	9.76	32
678	3.76	15.09	.006	.154	2.70	7.98	.248	9.50	34
686	4.62	20.00	.043	.157	6.95	15.7	.191	17.81	35
740	4.14	14.28	.005	.148	2.70	7.85	.252	8.88	39
745	4.14	14.21	.001	.148	2.82	6.95	.253	7.52	41
751	4.33	15.59	.018	.149	3.12	10.5	.232	12.30	36
815	3.22	13.80	.018	.153	3.12	10.5	.269	12.78	31
819	3.21	14.60	.002	.152	2.86	7.20	.253	8.17	31
822	3.31	14.96	.005	.153	2.70	7.85	.249	9.65	30
826	3.98	18.90	.043	.157	6.95	13.1	.202	17.88	32
829	3.14	14.35	-.005	.156	3.24	5.63	.264	5.73	37
878	2.17	13.37	-.013	.154	4.20	4.08	.280	2.52	47
883	2.17	13.80	-.008	.153	3.54	5.04	.271	4.34	31
888	1.89	14.30	-.004	.155	3.18	5.94	.264	6.07	22
892	3.27	16.08	.010	.158	2.86	7.62	.239	9.24	29
940	1.50	13.20	-.018	.157	4.92	3.00	.289	1.06	110
944	1.65	13.93	-.009	.156	3.66	5.76	.272	5.40	22
948	2.28	15.52	.002	.160	2.86	7.20	.251	8.75	21
980	1.01	12.52	-.017	.150	4.80	4.92	.291	2.95	23

NASA-Langley, 1967 —— 12 E-3476